Int J Theor Phys (2010) 49: 2506–2511
DOI 10.1007/s10773-010-0436-9

The Effect of Dark Matter on Solar System and Perihelion Precession of Earth Planet

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Received: 12 May 2010 / Accepted: 14 July 2010 / Published online: 5 August 2010

Abstract This paper visualizes effect of dark matter on solar system and especially perihelion precession of Earth planet. The relation between the rate of perihelion shift of Earth planet and dark matter are obtained.

Keywords Dark matter · Perihelion precession · Earth · Cosmology

1 Introduction

Until 1930, most of physicists and cosmologists believed that every material in the world is visible by unaided eye or by using any instrument, and since the electromagnetic waves are sent from everything, man could be able to see anything in the world using a detector of electromagnetic waves [1, 2]. But about 1930, two astronomers, Zwicky and Smith, in their study on two cosmetic clusters, named "Virgo" and "Coma Berenices", found out that their velocity was 10–100 times more than what was estimated.

In one cluster, the gravity force of galaxies is the most important factor affecting the velocity of galaxies, and (due to the Kepler's rules) velocity of a galaxy shows the amount of the existing material in that galaxy [3]. So there should be another material in the two mentioned clusters, hidden from our eyes (dark matter). This could be the strongest reason of existing of dark mass and energy.

In 1970, a more reliable reason was found due to the efforts of physicists in measuring the galaxies rotation graphs. The reason was that according to the Kepler's rules, velocity of rotation around a center depends on the distance of r from the center of the mass [$V = \sqrt{\frac{GM}{r}}$]. Therefore, the velocity of a galaxy is measurable, having the amount of mass and the distance and vice versa, knowing the velocity, one can measure the mass [4]. Scientists expected the rotation velocity to reduce at the edges of the galaxy, where the light decreases quickly [5].

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But it was observed that velocity didn't decrease and it was fixed. The observation proved that at the edge of a galaxy there are masses, which are not visible, astronomically [4]. A phenomenon must be seen directly or should be recognized indirectly through its signs and traces [1]. Every phenomena sending light or radio waves would be visible if being at zone of the radio telescope recipients [2]. But any phenomenon without this characteristic would not be visible physically [5, 6].

The observation had been done on some spiral galaxies such as our Milky Way galaxy. The results were the same, which could be the most important reason for existence of the missing mass and the dark energy [7]. Some cosmologists believe that dark matter is made of strange materials named WIMPs (Weak Interactive Massive Particle). WIMPs are a group of heavy particles which do not interact with other particles or the interaction is hardly accomplished. Axions are one example of these particles [8–10].

The effect of dark matter on solar system and perihelion precession motion and their relation will be discussed and analyzed in this study.

2 Perihelion Precession of Earth Planet

The long orbital elliptical diameter of Earth rotates in the space slowly (due to the perihelion precession). According to the Newton's gravitation theory (in classical mechanics), elliptical orbits of planets in space, in the absence of the effect of the external planets, are perfect, fixed elliptic (absolute space-time). The relativity theory of Einstein claims that the space curve affects the orbit of planets and the elliptical orbits of planets would be forced to rotate around the sun slowly. Newton's rules show the distance of Earth to Sun, according to the solar perihelion precession, as the following equation,

$$r = r_{\min}\left(\frac{1+e}{1+e\cos\varphi}\right), \tag{1}$$

where $r_{\min}$ is the distance of the perihelion precession of Earth and "e" is the eccentricity of planetary orbits, measured as follows,

$$e = \frac{c}{a}\sqrt{1-\frac{b^2}{a^2}}, \tag{2}$$

where a is the half of the long diameter of the elliptic, b is the half of the short diameter, and c is the distance between these two centers. According to Einstein's relativistic gravity equations, (1) transforms to the following equation:

$$r = r_{\min}\left(\frac{1+e}{1+e\cos(\varphi-\Delta\varphi)}\right), \tag{3}$$

where $\Delta\varphi$ is the measure of the perihelion of a planet in a whole rotation around the Sun, (with regard to Newton's equations) which is measured by the following relativistic equation,

$$\Delta\varphi = \frac{6\pi GM_{\Theta}}{ac^2(1-e^2)}, \tag{4}$$

where M_{Θ} is the mass of Sun, G is constant general gravity and a is the long semi-axis of Earth orbit.

3 Measuring the Extra Perihelion Precession of Mercury, Venus and Earth Planets

Astronomic observations show that while Mercury planet rotates around its orbit, its elliptical orbit is also rotating in the space (i.e. the elliptical surface of the orbit in space is not constant). The long elliptical diameter and, as a result, the perihelion of Mercury move 574 arc second per century in space [11]. If the motion observed from the solar system, it is anti-clockwise, and this is because of the gravity effect of Sun and other planets on Mercury, according to the classic physics rules, i.e. the phenomenon was explainable by Newton's gravity theory. But the theory expects the motion about 40–50 arc seconds per century less than what was observed and measured. To solve the difference, some physicists suggested that it would be better to manipulate the gravity rule of Newton. The matter was not explainable and measurable by Newton's classic mechanics rules, and in 1900s, astronomers and physicists did suffer from this problem, and were trying to find an explanation. Finally, in 1916, Albert Einstein solved the problem, presenting his general relativistic theory. The basic point of the theory was that the gravity field would appear through four-dimension time-space curve.

Einstein's general relativistic theory's equations predicted that the long diameter of elliptical orbit of Mercury should have an extra 43 arc seconds per century [5]. According to the measure of the extra perihelion precession motion of the planet in a whole rotation around the Sun (4) and by using the equation, the perihelion precession motion for Mercury, Venus, and Earth were about 42.57, 8.65, 3.83 arc seconds per century. The results exactly conform the astronomical observations (the observations showed that the measure of the extra precession for Mercury, Venus, and Earth are 43.1, 8.4, and 5.0).

Different experiments confirmed the above measurements and Einstein's general relativistic theory. Quantum relativistic physics and quantum gravity physics propound and analyze some wonderful things in this field [12].

4 The Relation Between Perihelion Precession of Earth Planet and Dark Matter

Now we are going to analyze the relation between perihelion precession motion of Earth and dark matter. The behavior of the observed rotation curve of Milky Way galaxy shows that there should be a large amount of a non-brilliant matter scattered in a wide spread space. So many rotation curves of other galaxies would also confirm the existence of the dark matter in other places in the universe. It can be proved that a symmetrical sphere distribution of a matter with the density of,

$$\rho(r) \approx \frac{1}{r^2} \tag{5}$$

will become a flat rotation curve,

$$G\frac{mM(r)}{r^2} = G\frac{m}{r^2}\int_0^r 4\pi r^2 \rho(r)dr. \tag{6}$$

The required matter for producing such a rotation curve is more than the visible and observable masses [13]. To determine how much of the whole mass of a galaxy is formed of the missing dark invisible matter is one of the questions in today's physics world. The density of a matter is inverse square distance ($\rho\ \frac{1}{r^2}$), therefore, the mass of the matter is equal to the radius distance,

$$\rho(r) = \frac{M}{V} = \frac{M}{r^3} \approx \frac{1}{r^2}. \tag{7}$$

In a long distance from the mass center, the density reduce faster than $\frac{1}{r^2}$, because otherwise, the galaxy mass goes to infinity. Many problems would have been solvable if it was possible to measure the rotation curve from long distances of a galaxy (in long distances the density of a visible matter is insignificant and it is impossible to measure the radius velocity by present observation facilities). Showing the density of the missing matter by ?dm and supposing it to be a symmetrical sphere, the center of which in solar system conforms to the Sun's center, mathematical analyzes of the perihelion precession motion of Icarus, which was used for the first time to improve a higher limit in the missing matter density [5] showed that the density zone was as follows:

$$\rho_{\mathrm{dm}} = 10^{-16}\ \frac{\mathrm{g}}{\mathrm{cm}^3}. \tag{8}$$

(Recently, a precise astronomical table, named EPM has been designed which shows the amount of deflections in theory calculations results of observation information and indicates the required corrections [7, 14].) But by analyzing the possible corrections of perihelion precession motion of Earth, Mars, and Mercury, the amount of the density of missing matter in solar system was as follows,

$$\rho_{\mathrm{dm}} = 10^{-19} \frac{\mathrm{g}}{\mathrm{cm}^3}. \tag{9}$$

The dark matter temperature also affects the above density, because using Jeans equation and mass density in the form of $\rho \sim r^{\alpha}$, one can determine a relation between temperature and density of dark matter as the following equation [15],

$$T_{\mathrm{dm}} = Cr^{\alpha} + \frac{1}{\alpha+1}\left(1 - \frac{2}{3}\beta_{\mathrm{dm}}\right)\frac{GM}{r}, \tag{10}$$

where β_{dm} is the only free parameter in dark matter anisotropy which can be treated as a constant. We could extract the dark matter temperature equations and define the permissible limits, explaining a model of matter density in three different limits [15].

Some advanced researches and analysis confirmed the results. Therefore, the potential energy equation around the dark matter would be as following,

$$dU(r) = -km \int \frac{M(r)}{r^2} dr. \tag{11}$$

Now, if the angular momentum of the planet denoted by τ, then the perihelion precession of the planet shift will be as follows [16],

$$\delta\theta = \frac{d}{d\tau}\frac{2m}{\tau}\int_0^{\pi} d\theta r^2 dU(r). \tag{12}$$

If eccentricity of planetary orbits (e) is small for a planet, then the relative perihelion precession motion of Earth per a time period will be as follows [13],

$$\frac{\delta\theta}{2\pi} = -\frac{2\pi\rho_{\mathrm{dm}}(r)r^3}{M_{\Theta}}, \tag{13}$$

where r is the radius of Earth rotation orbit ((14) is precise for a correction of (e^2), this correction for any planet in solar system is small).

Due to the fact that $V=\frac{4}{3}\pi r^3$, and $\rho_{\rm dm}=\frac{M(r)}{V}$ are produced from the constant density assumption, (13) can be rewritten as the following [5, 13, 17, 18],

$$\frac{\delta\theta}{2\pi}=-\frac{3}{2}\frac{M(r)}{M_\Theta}\sqrt{1-e^2}. \tag{14}$$

If we ignore the insignificant e^2, we will have,

$$\frac{\delta\theta}{2\pi}=-\frac{3}{2}\frac{M(r)}{M_\Theta}, \tag{15}$$

where $M(r)$ is mass of dark matter in a limited sphere with r radius. In other hands, according to following assumption,

$$\rho_{dm}=\rho_0\left(\frac{r}{r_0}\right)^{-\gamma} \tag{16}$$

(i.e. farther from the sphere center, the dark matter density will be smaller) we can also rewrite (13) as follows [13, 19, 20],

$$\frac{\delta\theta}{2\pi}=-\frac{3-\gamma}{2}\frac{M(r)}{M_S}. \tag{17}$$

The above equation shows that the more mass of dark matter is, the relative motion of perihelion precession of Earth per a time period will be more, i.e. perihelion precession rotation of Earth is conducted directly by a local characteristic of dark matter.

5 Conclusion

In this study the effect of dark matter on perihelion precession of Earth and Mars was explained scientifically and it was shown that the dark matter in solar system changes the rotation of planets orbits, e.g. according (17), the extra precession motion of Mercury and Earth is (42.57–0.00361), (3.83–0.00021) arc seconds per century. And the effect of dark matter on perihelion precession of Earth and Mars planets are (-0.00021 ± 0.00038) and (0.00011 ± 0.00047) arc seconds per century respectively.

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